

Math 2050, HW 2

- (1) Let $x_n = \sqrt{n+1} - \sqrt{n}$ for all $n \in \mathbb{N}$. Show that x_n and $\sqrt{n}x_n$ both converges.
- (2) (a) Suppose x_n is a sequence of positive number such that $\lim_{n \rightarrow +\infty} \frac{x_{n+1}}{x_n} > 1$. Show that x_n is a unbounded sequence.
(b) Suppose x_n is a sequence of positive number such that $\lim_{n \rightarrow +\infty} x_n^{1/n} < 1$, show that x_n is convergent. If the limit is 1, what can we conclude? Justify your answer.
- (3) (a) Let $x_1 = 1$ and $x_{n+1} = x_n + \frac{1}{x_n}$ for all $n \in \mathbb{N}$. Determine whether $\{x_n\}$ is convergent or not.
(b) Let $x_1 = 1$ and $x_{n+1} = \sqrt{2x_n}$ for all $n \in \mathbb{N}$, show that $\{x_n\}$ is convergent.
- (4) (Mistakes in previous version) If every subsequence of $\{x_n\}$ has a sub-subsequence converging to 0, show that $x_n \rightarrow 0$.